# Western Learning and Evidential Research in the Eighteenth Century *

Benjamin A. Elman
East Asian Studies and History
Princeton University

## Abstract

In the eighteenth century, classical scholars shared a simultaneous passion for antiquity and new forms of scholarship. Scholars re-appropriated the mathematical classics and early astronomy in the millennial quest for ancient wisdom. In a post-Jesuit world, the Ch'ing court during the Ch'ien-lung era was fortuitously buffered from contemporary European wars and the revolutionary changes then preoccupying Britain and France. In this geopolitical vacuum, Ch'ing literati sought to compare what they knew of European learning, brought principally by the Jesuits, with native learning. Though the priority was on the latter, the restoration of ancient learning allowed Manchus and Chinese to bring under control early modern European contributions in mathematics and astronomy.

The Jesuits in China had devised a unique, accommodation approach to gain the trust of the Ch'ing court and its gentry elites, which they rarely employed in Japan, India, or Southeast Asia, not to mention the "New World." Matteo Ricci and his immediate followers prioritized natural studies and mathematical astronomy during the late Ming and early Ch'ing because they recognized that Chinese literati and Ming and Ch'ing emperors were interested in such fields. Such literati interests in natural studies and "Western learning" continued in the eighteenth century despite the impact of the Rites Controversy. Hence, the account here challenges the usual image of Chinese lack of curiosity concerning early modern European science.

The reverse of current claims about Chinese disinterest in European science is the parallel assertion that Christianity and science had only marginal influence on Chinese literati before the nineteenth century. Many still emphasize the requirement to understand, first and foremost, the key, internal issues inscribed



* This essay is part of an ongoing project entitled "From the 'Chinese Sciences' (*Ko-chih hsüeh* 格致學) to 'Modern Science' in China (*K'o-hsüeh* 科學)" to be published by Harvard University Press.

in the classical debates of Ming-Ch'ing scholars. In the round, this claim has many merits. It overlooks, however, parallel events in European and Chinese intellectual and social history that imply that literati interests in European science were cut short not by Chinese disinterest but by the failure of the Jesuit mission to act as a reliable conduit of scientific and mathematical knowledge during and after the K'ang-hsi reign.

The Chinese "lack of knowledge" about scientific developments in eighteenth century Europe represented a breakdown of scientific transmission that can be tied directly to the demise of the Jesuits and their schools in Europe during the eighteenth century, which vicariously affected Chinese information about new trends there. Michel Benoist, for example, finally introduced an accurate account of Copernican cosmology in China after Church's ban ended in 1757. Anti-Jesuit polemics generated first by the Jansenists and later by the Enlightenment *philosophes*, however, led to suppression of the order, first in Portugal in 1759 and then by France, Spain, Naples, and Parma, before the Pope dissolved the order worldwide in 1773. China's "window on Europe" was shattered by forces internal to both European and Chinese history.



**Keywords**: Ch'ing dyasty, Science, Western learning

## Introduction

In the eighteenth century, classical scholars shared a simultaneous passion for antiquity and new forms of scholarship. Scholars re-appropriated the mathematical classics and early astronomy in the millennial quest for ancient wisdom. In a post-Jesuit world, the Ch'ing court during the Ch'ien-lung era was fortuitously buffered from contemporary European wars and the revolutionary changes then preoccupying Britain and France. In this geopolitical vacuum, Ch'ing literati sought to compare what they knew of European learning, brought principally by the Jesuits, with native learning. Though the priority was on the latter, the restoration of ancient learning allowed Manchus and Chinese to bring under control early modern European contributions in mathematics and astronomy.[1]

1 Compare Ming-hui Hu, "Provenance in Contest: Searching for the Origins of Jesuit Astronomy in Early Qing China, 1664-1705," *The International History Review*, v. 24, no. 1 (March 2002), pp. 1-36.

The Jesuits in China had devised a unique, accommodation approach to gain the trust of the Ch'ing court and its gentry elites, which they rarely employed in Japan, India, or Southeast Asia, not to mention the "New World." Matteo Ricci (1552-1610) and his immediate followers prioritized natural studies and mathematical astronomy during the late Ming and early Ch'ing because they recognized that Chinese literati and Ming and Ch'ing emperors were interested in such fields. Such literati interests in natural studies and "Western learning" continued in the eighteenth century despite the impact of the Rites Controversy. Hence, the account here challenges the usual image of Chinese lack of curiosity concerning early modern European science.

The reverse of current claims about Chinese disinterest in European science is the parallel assertion that Christianity and science had only marginal influence on Chinese literati before the nineteenth century. Many still emphasize the requirement to understand, first and foremost, the key, internal issues inscribed in the classical debates of Ming-Ch'ing scholars. In the round, this claim has many merits. This approach overlooks, however, parallel events in European and Chinese intellectual and social history that suggest that literati interests in European science were cut short not by Chinese disinterest but instead by the failure of the Jesuit mission to act as a reliable conduit of scientific and mathematical knowledge during and after the K'ang-hsi reign. The Jesuits did not transmit "modern science" to China.[2]

The Chinese "lack of knowledge" about eighteenth century scientific developments in Europe, notably Newtonian mechanics and continental calculus, represented a failure of scientific transmission that can be tied directly to the demise of the Jesuits and their schools in Europe during the eighteenth century, which vicariously affected Chinese information about new trends there. Michel Benoist (1715-1774), for example, finally introduced an accurate account of Copernican cosmology in China only after Church's ban on Copernican astronomy ended in 1757. Anti-Jesuit polemics generated first by the Jansenists and later by the Enlightenment *philosophes*, however, led to suppression of the order, first in Portugal in 1759 and then by France, Spain, Naples, and Parma, before the Pope dissolved the order worldwide in 1773. China's "window on Europe" was shattered by forces internal to both European and Chinese history.

2 Benjamin A. Elman, "Jesuit Scientia and Natural Studies in Late Imperial China," *Journal of Early Modern History: Contacts, Comparisons, Contrasts*, v. 6, no. 3 (Fall 2002), pp. 209-232.

## The Academy of Mathematics in Peking

When the French Jesuits arrived in China after 1689, they successfully created a legitimate place for themselves in the direct service of the Ch'ing ruler. In fact, they equated the K'ang-hsi emperor with their own "Sun King," Louis XIV. In addition to their missionary work, they hoped to introduce contemporary French science in China. For instance, the French mission's first superior, Jean de Fontaney (1643-1710), held the chair of mathematics at the Paris Jesuit college Collège Louis le Grand between 1676 and 1685 before leaving for China. Joachim Bouvet (1656-1730) hoped that the K'ang-hsi emperor would establish his own Academy of Science that would emulate the Academy of Sciences in Paris.[3]

The "Studio for the Cultivation of Youth" (*Meng yang chai* 蒙養齋) was established in the suburban Lofty Pavilion (*Yüan ming yüan* 圓明園) imperial garden in 1712-1713 for astronomical and mathematical work inside the court. The K'ang-hsi emperor recognized the need to continue to employ French Jesuits on the calendar despite his dissatisfaction with Rome's papal policies toward China after the Rites Controversy. He invited the French Jesuits to work for him as they worked for the French Academy while abroad.[4]

The K'ang-hsi emperor also molded his own court's Academy of Mathematics (*Suan hsüeh kua*n 算學館) on the model of the Parisian Academy of Sciences, but it was strategically named after the T'ang dynasty school for mathematics. The Academy was established in 1713 in the "Studio for the Cultivation of Youth" for calendrical work, but only Ch'ing literati and bannermen were appointed. No Jesuits were allowed in this inner coterie of imperial scholars, which included the third prince, Yin chih 胤祉 (1677-1732). This post-Rites Controversy policy ensured that the Jesuits would not be unduly influential in court mathematics.[5]



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3 Han Ch'i 韓琦, "Pai Chin ti I ching yen chiu ho K'ang hsi shih tai ti Hsi hsüeh Chung yüan shuo" 白晉的易經研究和康熙時代的西學中源說 (Bouvet's research on the Change Classic and the theory that Western learning originated in China), *Han hsüeh yen chiu* 漢學研究, v. 16, no. 1 (June 1998), pp. 185-201.

4 Horng Wann-sheng, *Li Shan-lan: the Impact of Western Mathematics in China During the Late 19th Century* (Ph.D. dissertation, New York: City University of New York. Dept. of History, 1991), pp. 16-17.

5 Catherine Jami, "From Louis XIV's Court to Kanxi's Court: an Institutional Analysis of the French Jesuit Mission to China (1688-1722), in Hashimoto Keizō... [et al.], eds., *East Asian Science: Tradition and Beyond* (Osaka: Kansai University Press, 1995), pp. 493-499.

The K'ang-hsi court sought to escape the dynasty's reliance on the Jesuits in calendrical matters. Li's group included Wang Lan-sheng 王蘭生, who was granted the highest civil service degree in 1721 by the emperor because of his mathematical abilities and called a "palace graduate in mathematical astronomy" (*ch'ou jen chin shih* 疇人進士). Wang then entered the "Studio for the Cultivation of Youth" where the French Jesuits helped him to translate the works included in the *Sources of Musical Harmonics and Mathematical Astronomy* (*Lü li yüan yüan* 律曆淵源) collectanea, which Mei Chüeh-ch'eng 梅瑴成 (d. 1763) and Ch'en Hou-yao 陳厚耀 (1648-1722) helped on.

In 1712, Ch'en had proposed a new compendium of European mathematics to replace the late Ming *Calendrical Studies of the Ch'ung chen Reign* (*Ch'ung-chen li shu* 崇禎曆書) inspired by the Jesuits. The result was the *Sources of Musical Harmonics and Mathematical Astronomy*, which included the *Collected Basic Principles of Mathematics* (*Shu li ching yün* 數理精蘊). In 1713, the K'ang-hsi emperor charged Mei Chüeh-ch'eng and Ch'en Hou-yao with supervising Ho Kuo-tsung 何國宗, Minggatu (Ming An-t'u 明安圖, d. 1763), and others to complete the project. The *Sources* was printed in 1723. This special group of mathematical and calendrical specialists included Wei T'ing-chen 魏廷珍 and others whom Mei Wen-ting 梅文鼎 (1633-1721) had trained before he died in 1721.[6]

The emperor recruited more that one hundred promising scholars to join the Academy of Mathematics regardless of their civil examination status. Mei Chüeh-ch'eng was made chief and Minggatu assistant editor for preparation of the *Collected Basic Principles of Mathematics*. In addition to those in the Academy of Mathematics who studied mathematics, astronomy, and music, a large number of instrument makers were also hired for the technical needs of the new academy. A team of fifteen calculators verified the computations based on the theoretical notions, mathematical techniques and applications, and numerical tables in the first part of the *Collected Basic Principles*.

Patterned after mathematical textbooks used in Jesuit colleges, the *Collected Basic Principles* introduced European algebra, while the last part had a section

6 Li Yan and Du Shiran, *Chinese Mathematics: a Concise History*, translated by John Crossley and Anthony Lun (Oxford: Clarendon Press, 1987), p. 218 ; Jean-Claude Martzloff, *A History of Chinese Mathematics*, translated by Stephen Wilson (New York: Springer-Verlag, 1997), pp. 218-219 ; Catherine Jami, "Learning Mathematical Sciences During the Early and Mid-Ch'ing," in Benjamin Elman and Alexander Woodside, eds., *Education and Society in Late Imperial China*, 1600-1900 (Berkeley: University of California Press, 1994), pp. 231, 238-240.

on logarithms to the base ten, which drew on European methods to compute decimal logarithms. The Chinese mathematics that informed the *Collected Basic Principles* included traditional equation methods (*fang ch'eng* 方程) and techniques for computing the sides of a right-angled triangle (*kou ku* 勾股), which were based on Mei Wen-ting's reinterpretation of traditional techniques to solve simultaneous linear equations.

A successor to the late Ming *Calendrical Studies of the Ch'ung-chen Reign*, this new and influential collectanea included: (1) the *Compendium of Observational and Computational Astronomy* (*Li hsiang k'ao ch'eng* 曆象考成); (2) the *Collected Basic Principles of Mathematics*; and (3) the *Exact Meaning of the Pitch-pipes* (*Lü lü cheng i* 律呂正義), which were all compiled in the "Studio for the Cultivation of Youth" starting in 1712. The collection was intended as a series of textbooks for the "Studio" and for students in the Imperial College's (*Kuo tzu chien* 國子監) own Academy of Mathematics.

After the *Collected Basic Principles of Mathematics* was printed in 1723, no other European mathematical works were introduced into China until after the Opium War (1839-1842). Notably missing in China was the European discovery of the more dynamic differential and integral calculus by both Leibniz and Newton, which had exceeded the static limits of Greek geometry and Islamic algebra. Moreover, the version of Euclid's *Elements of Geometry* in the *Collected Basic Principles* remained the official version until 1865.[7]

## The K'ang-hsi Era *Compendium of Observational and Computational Astronomy*

K'ang-hsi era efforts at reform culminated in the 1724 promulgation of the *Compendium of Observational and Computational Astronomy* and a sequel. The European astronomy in the *Compendium* was mostly a century old, but the sequel of 1742 (*Li hsiang k'ao ch'eng hou pien* 曆象考成後編) adapted more recent European discoveries, such as Kepler's elliptic orbits, to the ends of traditional calendar reform.[8]

Overall, the Ch'ing experts appointed by the K'ang-hsi emperor followed Mei Wen-ting's lead in rejecting Jesuit efforts to insinuate Christianity into their

7 Catherine Jami, "Western Influence and Chinese Tradition in an Eighteenth-Century Chinese Mathematical Work," *Historia Mathematica*, v. 15 (1988), pp. 311-331.

8 Nathan Sivin, "Copernicus in China," Colloquia Copernica II. Etudes sur l'audience de la theorie heliocentrique (Warsaw: Union Internationale d' Historie et Philosophie des Sciences, 1973), pp. 63-75, 89-92.

astronomy. Mei's mathematical work fit in with the court's efforts to have a calendar that would fuse European and Chinese techniques into a greater system. When the *Compendium of Observational and Computational Astronomy* was drafted in 1722 and promulgated in 1724, for instance, it followed European models, but it was prepared by Chinese in the court with only indirect Jesuit input.[9]

The early Ch'ing calendars produced by the Jesuits were based exclusively on European models, but the new system cobbled together in the *Compendium* fused European with "Chinese methods." Mei Chüeh-ch'eng and his group of native specialists affirmed Mei Wen-ting's efforts to reach a higher synthesis, which would supersede European and Chinese systems. During the 1720s, the court specialists mastered Jesuit astronomical methods and made them part of the imperial repertoire for computational astronomy.



However, Ch'ing specialists had no domestic incentive to go beyond the immediate needs of the Ch'ing calendar, now successfully reformed. Nor were they intellectually pressed by the Jesuits to do so. By 1725, the latter were themselves no longer on the cutting edge of the early modern sciences, and their mathematics went no further than simple algebra, trigonometry, and logarithms, which had been domesticated by a small group of late Ming and early Ch'ing specialists. In the eighteenth century, a larger community of Ch'ing classical scholars associated with evidential studies (*k'ao cheng hsüeh* 考證學) would restore traditional Chinese mathematics to a level of classical prestige.[10]

Although, the "Studio for the Cultivation of Youth" did not continue in the Yung-cheng era, nevertheless, the development of official studies in mathematics by selected bannermen, initiated by the K'ang-hsi emperor in 1670, was expanded in scope by the Yung-cheng emperor in 1734. In 1739, the Ch'ien-lung emperor placed mathematics as a field of study under the purview of the Dynastic School system. Han Chinese students outside the Astro-calendric Bureau (*Ch'in t'ien chien* 欽天監) could now study mathematics officially.

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9 Wang P'ing 王萍, "Ch'ing ch'u li suan yen chiu yü chiao yü" 清初曆算研究與教育 (Early Ch'ing research on Mathematical Astronomy and Education), *Chung yan yen chiu yüan chin tai shih yen chiu suo chi k'an* 中央研究院近代史研究所集刊, v. 3 (1972), p. 369.

10 Hashimoto Keizō 橋本敬造, "Rekisho Kōsei no seiritsu" 曆象考成の成立 (The formation of the compendium of observational and computational astronomy), in Yabuuchi Kiyoshi 藪內清 and Yoshida Mitsukuni 吉田光邦, eds., *Min Shin jidai no kagaku gijutsu shi* 明清時代の科學技術史 (History of Science and Technology in the Ming and Ch'ing periods) (Kyoto: Kyōto Daigaku Jinbun Kagaku Kenkyūjo, 1970), pp. 49-92.

Despite the Yung-cheng rejection of Jesuit learning in court, the key K'ang-hsi era advisors under the leadership of Mei Chüeh-ch'eng still played an influential role in the succeeding Ch'ien-lung court.[11]

When compared to eighteenth century developments in Europe, however, the fate of the Ch'ing dynasty Academy of Mathematics is instructive. In France, the Paris Academy of Sciences became a building block for an increase in science professionals and the institutions that supported them. Such institutional changes encouraged the eclipse of the more general learned societies and favored the rise of more specialized institutions. The establishment of professional standards for scientific disciplines by the late eighteenth century was accompanied by the expansion of universities and research institutes where professionalized science slowly incubated in institutions of higher learning, and specialized laboratories eventually replaced gentlemanly academies. Not until the late nineteenth century, would such developments commence in China.[12]



## Revival of Ancient Chinese Mathematics

Mei Chüeh-ch'eng lamented the destruction of Yüan-Ming astronomical instruments that had been in the Astro-calendric Bureau until 1672 when they were replaced by Ferdinand Verbiest's (1623-1688) new instruments. Mei had seen them in storage in 1713-1714, but in 1715 Bernard-Kilian Stumpf (1655-1720), then in charge of the Bureau, had several melted down to build a bronze quadrant. By 1744, only the armillary sphere, simplified sphere, and a celestial globe were left of the older instruments. Such material losses of the traditional calendrical heritage influenced Mei Chüeh-ch'eng's efforts to recover Sung-Yüan "single unknown" (*t'ien-yüan shu* 天元術) algebraic techniques for manipulating several unknowns, an enterprise that became a major mathematical feature of evidential research.

To this end, Mei focused on the Yüan minor official Li Yeh's 李冶 (1192-1279; originally Li Chih 李治) *Sea Mirror of Circular Measurement* (*Ts'e yüan hai ching* 測圓海鏡) of 1248, which was the oldest extant work on the "single unknown" technique. Under the Ming, the tradition of mathematical calculations

11 Wang P'ing, "Ch'ing ch'u li suan yen chiu yü chiao yü," pp. 370-371. See also Ming-hui Hu, *Cosmopolitan Confucianism: China's Different Road to Modern Science* (1664-1830) (Ph.D. dissertation, Los Angeles: UCLA, Dept. of History, 2003), chapter 3.

12 Roger Hahn, *The Anatomy of a Scientific Institution: the Paris Academy of Sciences, 1666-1803* (Berkeley: University of California Press, 1971), pp. 275-285.

associated with the *Computational Methods in Nine Chapters* (*Chiu chang suan shu* 九章算書) had been continued. However, the pioneering algebraic methods for solving polynomial equations developed by Ch'in Chiu-shao 秦九韶 (1202-61), Li Yeh, and Chu Yen-chieh 朱岩傑 (fl. end of 13th century) were not studied.[13]

**Recovery and Collation of Ancient Chinese Mathematical Works**

During the mid-Ch'ing revival of interest in mathematics, Mei Chüeh-ch'eng and others also realized that they no longer had access to many of the works originally included in the medieval *Ten Computational Classics (Shih pu suan ching* 十部算經). Moreover, in addition to Li Yeh's *Sea Mirror*, the seminal works of Ch'in Chiu-shao on polynomial algebra had been unavailable in Mei Wen-ting's time. In the midst of the "closed door" policies of the Yung-cheng emperor and his successors a large-scale effort to recover and collate the treasures of ancient Chinese mathematics became a major aspect of the late eighteenth and early nineteenth century internalist turn in evidential studies.[14]



In addition to the more famous evidential scholars such as Tai Chen 戴震 (1724-1777), Ch'ien Ta-hsin 錢大昕 (1728-1804), Juan Yüan 阮元 (1764-1849), and Chiao Hsün 焦循 (1763-1820), who stressed mathematics in their research, the editing of ancient mathematical texts and the continued digesting of European mathematical knowledge was carried out by a series of literati mathematicians who were also active in evidential studies:

Ch'en Shih-jen 陳世仁 (1676-1722)
Minggatu (Ming An-t'u)
Li Huang 李潢 (d. 1811)
Wang Lai 汪萊 (1768-1813)
Li Jui 李銳 (1773-1817)
Hsiang Ming-ta 項名達 (1789-1850)
Shen Ch'in-p'ei 沈欽裴 (n.d.)
Lo Shih-lin 羅士琳 (1789-1853)
Tung Yu-ch'eng 董祐城 (1791-1823)
Tai Hsü 戴煦 (1805-1860)
Li Shan-lan 李善蘭 (1811-1882)[15]

Many of the collations of mathematical texts were carried out under

13 Arthur Hummel, ed., *Eminent Chinese of the Ch'ing Period* (Washington: U.S. Government Printing Office, 1943), hereafter, ECCP, p. 569 ; Martzloff, *A History of Chinese Mathematics*, p. 20.

14 Elman, "Geographical Research in the Ming-Ch'ing Period," *Monumenta Serica*, v. 35 (1981-83), pp. 1-18.

15 Li and Du, *Chinese Mathematics*, pp. 223-224.

imperial auspices during the last years of the K'ang-hsi reign when the *Synthesis of Books and Illustrations Past and Present* encyclopedia (*Ku chin t'u shu chi ch'eng* 古今圖書集成) was completed. When published under Yung-cheng in 1726, it included some European calendrical and mathematical texts from the late Ming *Calendrical Studies of the Ch'ung-chen Reign* in its Ch'ing version known as the *Calendrical Studies According to New Western Methods* (*Hsi yang hsin fa li shu* 西洋新法曆書). Also included in the calendar section of the encyclopedia were five works of ancient and traditional Chinese mathematics:

1. *Chou Dynasty Classic of Gnomonic Computations* (*Chou pi suan ching* 周髀算經)
2. *Notes on Bequeathed Mathematical Arts* (*Shu shu chi i* 數術記遺)
3. *Mathematical Manual of Hsieh Ch'a-wei* (*Hsieh Ch'a-wei suan ching* 謝察微算經)
4. Mathematics in the *Brush Talks from the Dream Brook* (*Meng hsi pi t'an* 夢溪筆談)
5. *Systematic Treatise on Computational Methods* (*Suan fa t'ung tsung* 算法統宗)

When the first set of the Ch'ien-lung Imperial Library collection was completed between 1773 and 1781, its compilers also included several classical collators as well-versed in mathematics as Tai Chen: K'ung Chi-han 孔繼涵 (1739-84), Ch'en Chi-hsin 陳際新 , Kuo Ch'ang-fa 郭長發, and Ni T'ing-mei 倪廷梅. The "astronomy and mathematics" (*T'ien wen suan fa* 天文算法) category incorporated 58 works into the collection (see below). Several older, lost mathematical texts were recopied from the early Ming *Great Compendium of the Yung-lo Reign* (*Yung lo ta tien* 永樂大典), which had survived in the imperial court relatively intact. The general catalog of the Imperial Library, for example, included twenty-five notices on mathematics. Of these, nine were on the T'ang Computational Classics, three were for Sung-Yüan works, four on works from the Ming period, including the Ricci and Li Chih-tsao 李之藻 (1565-1630) partial translation of Euclid's *Elements*, and nine on works from the Ch'ing, most importantly the *Collected Basic Principles of Mathematics* (*Shu li ching yün*) and several works by Mei Wen-ting.[16]



16 *Ssu k'u ch'üan shu tsung mu* 四庫全書總目 (Catalog of the complete collection of the four treasuries), compiled by Chi Yün 紀昀... [et al.] (reprint, Taipei: I wen yin shu kuan, 1974), hereafter SKCSTM, chapters (*chüan*) 106-107 ; Martzloff, *A History of Chinese Mathematics*, pp. 32-33 ; ECCP, p. 637.

Eighteenth century efforts to recover ancient mathematical works extended beyond the borders of the Ch'ing dynasty. The role of Korea and Japan in preserving lost Chinese works is generally well-known. Worthy of special mention in this regard, however, was Juan Yüan's recovery of the lost *Primer of Mathematical Calculations* (*Suan hsüeh ch'i meng* 算學啓蒙) by Chu Shih-chieh from a 1660 Korean edition. It had been used as textbook in Korea during the fifteenth century after it was reprinted there in 1433.[17]

Published in 1299 in Yang-chou, Chu Shih-chieh's work described the rudiments of polynomial algebra. Several Korean emissaries stayed in Peking in the early nineteenth century when Juan Yüan's scholarship was influential there, particularly Juan's work on ancient technology entitled *Explications Using Diagrams of the Design of Wheeled Carriages in the "Artificer's Record"* (*K'ao kung chi ch'e chih t'u chieh* 考工記車制圖解). Later Kim Chông-hui 金正熹 (1786-1856) visited Peking in 1809 and met Juan Yüan in 1810. After their meeting, Kim sent Juan the Korean edition of the *Primer of Mathematical Calculations*, and Juan reciprocated bysending a number of his works to Kim. Juan and others were interested in Chu Shih-chieh's role in the formation of "single unknown" methods.[18]

Moreover, as the Ten Computational Classics were reconstituted, the Sung-Yüan works of Ch'in Chiu-shao, Chu Shih-chieh and Li Yeh, among others, also reappeared. A special, rare edition of seven of the Ten Computational Classics was reprinted by the Imperial Printing Office, including the *Chou Dynasty Classic of Gnomonic Computations*, and *Computational Methods in Nine Chapters*, as well as 100 chapters (*chüan* 卷) from the *Sources of Musical Harmonics and Mathematical Astronomy* of the K'ang-hsi era. Traditional mathematical works were also reprinted in several collectanea such as the *Ripple Pavilion Collectanea* (*Wei p'o hsieh ts'ung shu* 微波榭叢書), the *Collectanea from the Can't know Enough Pavilion* (*Chih pu tsu chai ts'ung shu* 知不足齋叢書), and the *Collectanea of the I-chia Hall* (*I chia t'ang ts'ung shu* 宜稼堂叢書).[19]



17 Satō Ken'ichi, "Re-evaluation of *Tengenjutsu* or *Tianyuanshu*: in the context of comparison between China and Japan," *Historia Scientiarum*, v. 5, no. 1 (1995), pp. 57-67.

18 Lam Lay-Yong, "Chu Shih-chieh's 'Suan hsüeh ch'i meng' (Introduction to Mathematical Studies)," *Archive for History of Exact Sciences*, v. 21, no. 1 (1979), pp. 1-31 ; Fujitsuka Chikashi 藤塚鄰, *Nichi Sen Shin no bunka kōryū* 日鮮清の文化交流 (Cultural exchange between Japan, Korea, and Ch'ing China) (Tokyo: Chūbunkan shoten, 1947), p. 77.

19 Li and Du, *Chinese Mathematics*, pp. 225-226.

**Reconstruction of the Ten Computational Classics**

In the late Ming, Hsü Kuang-ch'i 徐光啓 (1562-1633) had claimed in his preface to Ricci's *Translations of Guidelines for Practical Arithmetic* (*T'ung wen suan chih* 同文算指) that the *Ten Mathematical Classics* were inferior to Jesuit mathematics. As a result of the recovery and collation of ancient mathematical texts, Hsü's claims about the superiority of Jesuit mathematics were increasingly disparaged by evidential scholars who appealed to the "Chinese origins of Western Learning" as a historical reality and not just a political tactic to justify calendrical reform, as had been the case for Hsü in the last years of the Ming.[20]

Indeed, only the *Chou Dynasty Canon of Gnomonic Computations* was still printed and available widely in Ming times. The rest of the mathematics canon were either lost or not available to scholars. The sources left by the late Ming were derived from Southern Sung editions or from the early Ming *Great Compendium of the Yung lo Era*. Fortunately in the late Ming, Mao Chin 毛晉 (1599-1659) and Mao I 毛扆 (1640-ca. 1710) collated seven of the mathematical classics from Southern Sung editions for their Su-chou printing house known as the "Pavilion Reaching to the Ancients" (Chi ku ko 汲古閣):

1. *Chou Dynasty Canon of Gnomonic Computations* (*Chou pi suan ching*)
2. *Sun-tzu's Computational Canon* (*Sun tzu suan ching* 孫子算經)
3. *Computational Canon of the Five Administrative Departments* (*Wu ts'ao suan ching* 五曹算經)
4. *Chang Ch'iu-chien's Computational Canon* (*Chang Ch'iu-chien suan ching* 張邱建算經)
5. *Computational Canon of the Continuation of Ancient Techniques* (*Chi ku suan ching* 輯古算經)
6. *The Marquis of Hsia, Yang's Computational Canon* (*Hsia hou Yang suan ching* 夏侯陽算經)
7. an incomplete version of the *Computational Methods in Nine Chapters* (*Chiu chang suan shu*)

Although criticized for their numerous errors due to some slipshod xylography, the "Pavilion Reaching to the Ancients" versions of the Classics and the Dynastic Histories were highly prized. Among Mao's specialties was a



20 Roger Hart, "Xu Guangqi, Memorialist," presented at the Colloquium sponsored by the Center for the Cultural Studies of Science, Medicine, and Technology, Dept. of History, UCLA (April 15, 2002).

process for making facsimiles of Sung editions by tracing every feature of the rare books he borrowed from other collectors. The *Chou Dynasty Canon* and *Notes on Bequeathed Mathematical Arts* (*Shu shu chi i* 數術記遺) were also published in a Wan-li era (1573-1619) collection.[21]

Later, however, the "Pavilion Reaching to the Ancients" editions were dispersed and fell into book collectors' hands. In the process, only five of the ten mathematical classics were intact in the early Ch'ing. A manuscript copy of the *Computational Methods in Nine Chapters* was sent to the K'ang-hsi court and kept in an imperial pavilion. Subsequently, the collating of the "Ten Computational Classics" was accelerated upon the 1728 publication of the *Chou Dynasty Canon of Gnomonic Computations* and *Notes on Bequeathed Mathematical Arts*, both in the *Synthesis of Books and Illustrations Past and Present* encyclopedia. The notoriety that Mei Wen-ting had achieved as a mathematician, coupled with the publication of several new European mathematical works during the late K'ang-hsi reign brought mathematical astronomy into the mainstream of classical studies.



While serving on the Imperial Library commission in the 1770s, Tai Chen collated seven of the ten mathematics classics from the *Great Compendium of the Yung-lo Era*. In addition, he recovered two more from manuscript copies originally held by the Mao family, which were published in the Imperial Printing Office Collectanea of Rare Editions (*Wu ying tien chü chen pan ts'ung shu* 武英殿聚珍版叢書). Tai's colleague K'ung Chi-han had them reprinted in the *Ripple Pavilion Collectanea* in 1773 under the title Ten Mathematical Classics (*Suan ching shih shu* 算經十書). Subsequent editions were based on these late Ch'ien-lung versions.[22]

The rediscovery and reconstruction of the mathematical classics stimulated interest in them, and they were increasingly studied by evidential scholars such as Tai Chen, Li Huang, Shen Ch'in-p'ei, and Ku Kuan-kuang 顧觀光. They produced several important works on mathematical texts in the follow-up style of scholarship known as "additions and corrections" (*pu cheng* 補正) in Ch'ing dynasty classical book titles. Li Huang, for example, prepared works entitled *Careful Examination with Diagrams of the Computational Methods in Nine Chapters* (*Chiu chang suan shu hsi ts'ao t'u sho* 九章算術細草圖說), *Careful*

21 Martzloff, *A History of Chinese Mathematics*, p. 125.

22 Li and Du, *Chinese Mathematics*, pp. 226-227.

*Examination with Diagrams of the Sea island Computational Canon* (*Hai tao suan ching hsi ts'ao t'u shuo* 海島算經細草圖說), and *Examination of Annotations of the Ancient Mathematical Classics* (*Chi ku suan ching k'ao chu* 緝古算經考注). Ku Kuan-kuang, for instance, published his *Collation Notes for the Chou Dynasty Canon of Gnomonic Computations* (*Chou pi suan ching chiao k'an chi* 周髀算經校勘記).[23]

**Recovery of Sung-Yüan Mathematical Works**

Scholarly efforts in the late Ch'ien-lung era to reconstruct the "single unknown" and "four unknowns" (*Ssu yüan shu* 四元術) techniques for solving complex equations in several unknowns and to several powers, led to increased collation and research on the mathematical texts produced by a long neglected group of Sung-Yüan minor officials and commoner mathematicians. Ch'in Chiu-shao's *Computational Techniques in Nine Chapters* (*Shu shu chiu chang* 數術九章, 1247), for example, provided general algorithms for solving simultaneous equations and remainder problems (*Ta yen ch'iu i shu* 大衍求一術, "great expansion procedure for finding 1"). His method was similar to the Horner/Ruffini rule devised in the early nineteenth century, which invented a technique for the numerical calculation of the roots of polynomial equations. In the *Computational Techniques*, Ch'in used a parallel technique of alternating additions or subtractions, a procedure also used by Yang Hui 楊輝 (fl. in Hang-chou during the Southern Sung), Chu Shih-chieh, and others.[24]

Although Ch'in's work followed somewhat the overall structure of the *Computational Methods in Nine Chapters*, his algorithms were much more sophisticated. In addition, because Ch'in had studied in the Sung Astro-calendric Bureau in Hang-chou as a youth, his work also dealt with calendrical chronology as a problem in remainder theory. Finally, the *Computational Techniques* calculated the area of an arbitrary triangle as a function of the lengths of its three sides, which was similar to the proto-trigonometric relational features for computing the sides of a right-angled triangle (*kou ku*). Later, this approach drew



23 Li and Du, *Chinese Mathematics*, pp. 227-230 ; Elman, *From Philosophy to Philology: Social and Intellectual Aspects of Change in Late Imperial China* (Second edition) (Los Angeles: UCLA Asian Pacific Monograph Series, 2001), pp. 242-244.

24 Ch'ien Pao-ts'ung 錢寶琮, *Ch'ien Pao-ts'ung k'o hsüeh shih lun wen hsüan chi* 錢寶琮科學史論文選集 (Selected essays on history of Science by Ch'ien Pao-ts'ung) (Peking: K'o hsüeh ch'u pan she, 1983), pp. 22-36 ; Alexander Wylie, *Notes on Chinese Literature* (reprint ed., Taipei: Bookcase Shop Limited, 1970 ; original ed., Shanghai: American Presbyterian Press, 1867), p. 116 ; Martzloff, *A History of Chinese Mathematics*, pp. 149-152.

the attention of Tai Chen and others interested in equating such proto-trigonometric relational features with Jesuit trigonometry.[25]

Ch'in's *Computational Techniques* was copied from the *Great Compendium of the Yung-lo Era* into the Ch'ien-lung Imperial Library by Tai Chen. Chiao Hsün and Li Jui each studied and wrote on Ch'in's findings. Later Shen Ch'in-pei discovered a Ming manuscript of the *Computational Techniques*, which he compared to the version in the *Great Compendium of the Yung-lo Era*. A definitive edition of the *Computational Techniques* was then included by in the 1842 *Collectanea of the I chia Hall* and became the basis for the modern versions. Hua Heng-fang's 華蘅芳 (1833-1902) *Mathematical Notes* (*Hsüeh suan pi t'an* 學算筆談) for 1882-1888 presented a list of must-read books, which included both Chinese and Western works. Although he was well-versed in modern mathematics, Hua still recommended the *Computational Techniques* for solving remainder problems.[26]

After the Mongols conquered north China in 1232, Li Yeh prepared two works: the *Sea Mirror of Circle Measurement* (*Ts'e yüan hai ching*, 1248) and the *Adding to Ancient Techniques for Computing Geometric Figures* (*I ku yen tuan* 益古演段, 1259), which were copied into the Imperial Library. Although Li lived in reclusion in Shan-hsi after the Mongol triumph, he was called to the Mongol court to consult on governance and earthquakes. The *Sea Mirror* survived from a book in Li Huang's private library. The *Adding to Ancient Techniques* was copied into the *Compendium of the Yung-lo Reign*. Liu Jui later collated both, and they were also printed in the *Collectanea from the Can't know enough Pavilion*. Li Jui's edition of the *Sea Mirror* also included a series of explanatory notes.[27]

Although Yang Hui's mathematical works were all included in the *Compendium of the Yung-lo Reign*, they were not copied by Tai Chen into the Imperial Library. Part of Yang's *Continuation of Ancient Mathematical Methods for Elucidating Strange Numbers* (*Hsü ku chai ch'i suan fa* 續古摘奇算法) was,



25 Ulrich Libbrecht, *Chinese Mathematics in the Thirteenth Century: the Shu-shu Chiu-chang of Ch'in Chiu-shao* (Cambridge: MIT Press, 1973), passim, and Martzloff, *A History of Chinese Mathematics*, pp. 2-12, 231-247.

26 Wylie, *Chinese Literature*, p. 116 ; Hu Mingjie, *Merging Chinese and Western Mathematics: The Introduction of Algebra and the Calculus in China, 1859-1903* (Ph.D. dissertation, Princeton: Princeton University. Dept. of History, 1998), pp. 252-253.

27 Wylie, *Chinese Literature*, pp. 116-117.

however, edited for the *Collectanea from the Can't know enough Pavilion* and printed by the Hang-chou bookman Pao T'ing-po 鮑廷博 (1728-1814). In 1840, Yang's commentary and supplement for the *Computational Methods in Nine Chapters* and the *Yang Hsi's Calculation Methods* were included in the *Collectanea of the I-chia Hall*. Yang's complete work was lost in China but was rediscovered in the Ch'ing by Li Jui. In the early twentieth century, a 1433 Korean edition of the *Yang Hui's Calculation Methods*, based on a 1378 Ming edition, was found in Japan.[28]

On the other hand, Chu Shih-chieh's 1303 *Jade Mirror of the Four Unknowns* (*Ssu yüan yü chien* 四元玉鑒) and his 1299 *Primer of Mathematical Calculations* (see above) were not recovered in time to be included in the Imperial Library. Although the focus in Chu's *Jade Mirror* was on practical issues dealing with architecture, finance, military logistics, etc., it energized late Ch'ing evidential scholars who found in it a Chinese algebra to extract roots (*k'ai fang* 開方) predating the Jesuits. Chu's polynomial equations went beyond the second and third degrees up to the fourteenth.



During the Chia-ch'ing reign (1796-1820), Juan Yüan obtained a version of the *Primer of Mathematical Calculations* from a Korean envoy while governor of Chekiang, which he used to reconstitute the *Jade Mirror of the Four Unknowns*. He then sent it to Peking to be included as supplement for the Imperial Library. Juan also gave Li Jui a copy to collate, which Li left uncompleted. Others such as Hsü Yü-jen 徐有仁 (1800-1860) and Shen Ch'in-p'ei worked on it, although Shen's commentary was never published.[29]

Lo Shih-lin obtained a copy of the *Jade Mirror* in 1822, and after ten years of collation he presented a definitive reconstruction of "four unknowns" techniques, which he entitled the *Jade Mirror of the Four Unknowns with Detailed Calculations* (*Ssu yüan yü chien hsi ts'ao* 四元玉鑒細草) and had it printed in 1843 in Yang-chou. After the rediscovery of Chu Shih-chieh's *Primer of Mathematical Calculations*, Lo Shih-lin also had it reprinted in 1839 based on the Korean edition of 1660 recovered by Juan Yüan. As noted above, the *Primer*

28 Li and Du, *Chinese Mathematics*, pp. 230-231 ; Martzloff, *A History of Chinese Mathematics*, pp. 149-152, 157-159 ; Lay-Yong Lam, *A Critical Study of the Yang Hui Suan Fa, a Thirteenth-Century Mathematical Treatise* (Simgapole:Singapore University Press, 1977).

29 Jock Hoe, "Zhu Shijie and his 'Jade Mirror of the Four Unknowns'," in *First Australian Conference on the History of Mathematics: Proceedings of a Conference at Monash University* (November 1980: Clayton, Australia), no. 6 & 7, p. 105.

was also an important clue to the fundamentals of the "single unknown" and "four unknowns" polynomial algebra developed by Ch'in Chiu-shao and Li Yeh.[30]

Chang Tun-jen 張敦仁 in 1801 worked out every problem in the *Computational Canon of the Continuation of Ancient Techniques* based on "single unknown" procedures. Chang also prepared an 1831 work entitled *Technique for Finding "1"* (*Ch'iu i suan shu* 求一算術), i.e., indeterminate analysis based on the great expansion [*ta yen* 大衍] method for solving simultaneous congruencies. Li Jui, who was Chang's personal secretary, helped complete this harvest of Sung-Yüan mathematics and bring it into the mainstream of evidential studies in the first half of the nineteenth century.

The "internal turn" to native mathematics had enabled those Chinese classical scholars at the cutting edge of evidential studies to grasp the importance of advanced algebraic techniques for solving complicated equations based on sophisticated mathematical problems. When the differential (*wei fen* 微分) and integral (*chi fen* 積分) calculus was finally introduced by Alexander Wylie (1815-1887) and John Fryer (1839-1928) in the middle of the nineteenth century, its sophistication was readily appreciated by Li Shan-lan and others who had already mastered "single unknown" and "four unknowns'" problem-solving skills.[31]



## The Critique of Western Learning in the Ch'ien-lung Imperial Library

Biased in favor of Han Learning (*Han hsüeh* 漢學) and evidential research, the editors of the ambitious Ch'ien-lung Imperial Library project (*Ssu k'u ch'üan shu* 四庫全書) initiated a wellpublicized empire-wide search in the 1770s for every written work in the empire. They then engaged in a critical review of every book available to them, selected books worthy of inclusion in the collection, and carefully collated the final versions chosen for inclusion. Leading classical scholars in various fields were appointed to evaluate and collate books in their respective specialties. More than 360 scholars officially worked at the apex of a staff of several thousand in Peking.

30 Lo Shih-lin 羅士琳, *Ssu yüan yü chien hsi ts'ao* 四元玉鑑細草 (Reprint of 1836 ed., Shanghai: Shang wu yin shu kuan, 1937), chüan 1 ; Li and Du, *Chinese Mathematics*, pp. 115-117, 231-232 ; Martzloff, *A History of Chinese Mathematics*, pp. 153-157.

31 Wylie, *Notes on Chinese Literature*, pp. 115-116 ; Li and Du, *Chinese Mathematics*, pp. 242, 251.

The editors, many of them partisans of Han Learning, separated the "investigation of things and extension of knowledge" (*ko wu chih chih* 格物致知) from its longstanding association with Ch'eng-Chu 程朱 Sung Learning (*Sung-hsüeh* 宋學) and connected it instead with their emphasis on verifiable knowledge derived from empirically based research. They contended, for instance, that Ch'en Yüan-lung's 陳元龍 (1652-1736) early eighteenth century encyclopedic entries in the *Mirror of Origins Based on the Investigation of Things and Extending Knowledge* (*Ko chih ching yüan* 格致鏡原) "were all [examples of] broad learning and thus the work was entitled using the phrase for 'investigating things and extending knowledge.'"[32]

In many cases, the editors elided any mention of European learning when they could. For example, their summary of Fang I-chih's (1611-1671) late Ming *Notes on the Principles of Things* (*Wu li hsiao chih* 物理小識), which generally accepted Jesuit explanations of natural phenomena, presented Fang's "material investigations" in light of encyclopedias compiled since medieval times in China. The final version of the Imperial Library account made no effort to mention the notions of a spherical earth, limited heliocentrism, or human physiology that Fang had culled from Jesuit translations.[33]



The editors could not ignore the translations and other works that had been compiled in Chinese since the late Ming by the Jesuits and their collaborators. The Jesuit role in the K'ang-hsi era *Compendium of Observational and Computational Astronomy* and its supplement were addressed in the *General Catalog of Works in the Imperial Library* (*Ssu k'u ch'üan shu tsung mu* 四庫全書總目). The prestigious catalog mentioned thirty-six European works, twenty of which were copied into the Library. All were on natural studies.

On the other hand, the ten works from the "applications" (*ch'i* 器, lit., "implements") section of Li Chih-tsao's *First Collection of Celestial Studies* (*T'ien hsüeh ch'u han* 天學初函) were included, but not those from the "principles" (*li* 理) section, except for a geographical work by Giulio Aleni (1582-1649). Li Chih-tsao's 1628 *First Collection* placed the natural theology of the Jesuits under "principles" and material studies under "applications." Similarly, only two of Verbiest's works were given consideration in the Imperial

32 SKCSTM, 136.25a-26a ; R. Kent Guy, *The Emperor's Four Treasuries: Scholars and the State in the Late Ch'ien-lung Era* (Cambridge, Mass.: Council on East Asian Studies, Harvard University, 1987).

33 SKCSTM, 122.29a-b.

Library. The Library catalogers included mention of Aleni's *A Summary of Western Learning* (西學凡), but they downplayed it as a heterodox study and did not copy it into the Imperial Library.[34]

The editorial summary of Aleni's work presented the six fold classifications of the sciences that corresponded to sixteenth century European standards of learning. Seeing some parallels with native traditions, the editors analogized the Jesuit field of rhetoric (*wen k'o* 文科), which included grammar, history, poetry, and the art of writing, to the native category of "lesser learning" (*hsiao hsüeh* 小學). They likened philosophy (*li k'o* 理科), which for Jesuits included logic, physics, metaphysics, mathematics, and ethics, to classical teachings in the Great Learning. In addition, the editors praised the system of knowledge in Aleni's *Summary* and interpreted it light of the "investigation of things and fathoming principles" (*ko wu ch'iung li* 格物窮理). For Aleni the investigation of things to fathom principles was used as the Chinese term for contemporary European philosophy.[35]



Despite such correspondences, however, the Ch'ien-lung editors linked the teachings of the Great Learning to an emphasis on verifiable knowledge derived from empirical studies. Moreover, the editors concluded that Aleni had forced equivalences between European and native learning in order to substantiate Christianity as an ancient teaching in China, which late Ming scholars under the influence of Wang Yang-ming's 王陽明 (1472-1528) "studies of the mind" (*hsin hsüeh* 心學) had failed to reveal:

> "They also investigate things to exhaust principles, and seek to understand substance for practical use; in that they are roughly similar to our literati. However, the things they investigate are mostly petty, and the principles they seek to exhaust are mostly esoteric and untestable. That is why this book is considered heterodox."

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34 SKCSTM, 106.28a-36a ; Chi Wen-te 計文德, *Ts'ung Ssu k'u chüan shu t'an chiu Ming Ch'ing chien shu ju chih Hsi hsüeh* 從四庫全書探究明清間輸入之西學 (Inquiring into the importation of Western learning during the Ming-Ch'ing transition from the point of view of the Complete Collection of the Four Treasuries) (Taipei: Han Mei t'u shu yu hsien kung ssu, 1991), pp. 404-436.

35 SKCSTM, 106.1a-51a, 107.23a-24a, 125.27b-35b, 134.10a-11b ; Nicolas Standaert, S.J., "The Investigation of Things and the Fathoming of Principles (*Ko wu ch'iung li*) in the Seventeenth-Century Contact Between Jesuits and Chinese Scholars," in John W. Witek, S.J., ed., *Ferdinand Verbiest (1622-1688): Jesuit Missionary, Scientist, Engineer and Diplomat* (Nettetal: Steyler Verlag, 1994), pp. 412-417.

On this basis, the editors also ridiculed Alfonso Vagnoni's (1566-1640) 1633 *Treatise on the Composition of the Universe* (*K'ung chi ko chih* 空際格致) because it had tried to introduce a new reading for *ch'i* 氣 (= Aristotelian "air") that favored the ancient Greek notion of the four elements.[36]

**Astronomy and Mathematics in the Ch'ing Imperial Library**

The *General Catalog of Works in the Imperial Library*, completed in 1782, attempted to present the gist of a book in outline form so that readers could get a general idea of its contents. Except for entries related to the Jesuits, such accounts usually gave the reader an accurate and concise idea of the nature and importance of the work. Even though the editors chose to elide or ridicule Jesuit works that were included in the *General Catalog*, they did innovate when they presented surviving colophons on works dealing with astronomy and mathematics (*t'ien wen suan fa* 天文算法).

Tai Chen worked for the Imperial Library commission on the mathematical, astronomical, and calendrical texts included in the "Astronomy and Mathematics" section, which we have seen above enabled him to recover several of the Ten Computational Classics. Of the ten thousand titles reviewed by the editors about one-third finally were copied into the imperial manuscript collection, including fifty-eight works on "Astronomy and Mathematics." Forty of the latter were recovered by Tai Chen and his colleagues from the *Great Compendium of the Yung-lo Era*.[37]

The late eighteenth century classification of knowledge reveals the manner in which the variety of learning was perceived at that time. We also see the nature and structure of the concepts used to order that variety within the emerging disciplines in Ch'ing classical scholarship. The bibliographic clustering of subjects in the Ch'ien-lung Imperial Library in particular presents the culturally conditioned biases in Ch'ing Han and Sung Learning. Moreover, the eighteenth century structure of knowledge shaped evidential studies and influenced how new research on mathematical astronomy would be understood.



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36 SKCSTM, 125.31b-34a, 125.34b-35a. Compare Erik Zürcher, "Renaissance rhetoric in late Ming China: Alfonso Vagnoni's Introduction to his *Science of Comparison*," in Federico Masini, ed., *Western Humanistic Culture Presented to China by Jesuit Missionaries (XVII-XVIII centuries)* (Rome: Institutum Historicum S.I., 1996), pp. 331-359.

37 Chi Wen-te, *Ts'ung Ssu k'u chüan shu t'an chiu Ming Ch'ing chien shu ju chih Hsi hsüeh*, pp. 410-426.

Representing the classical scheme of disciplines in the late eighteenth century, the Imperial Library was based on the "four classifications system" (*ssu pu* 四部), which incorporated mathematical astronomy and calendrical studies, as well as medicine, as subcategories under the pre-Han "Masters" (*tzu pu* 子部) main category (see Table 1). Similarly the mathematical aspects of music were subsumed under the Classics, while chronography and geography were listed under the main category of History.[38]

The editors rejected placing mathematics in the "lesser studies" (*hsiao hsüeh*) section, which the late seventeenth century compilers of the *Ming History* had done, in favor of what they regarded as a more commonsense linking of mathematics to astronomy. Because of their association with esoteric and exotic fields such as astrology, chronomancy, the five phases, milfoil divination, prognostication, and geomancy, the editors separated the "numerological arts" (*shu shu* 數術) from their traditional association with mathematics and granted them an independent status.[39]

Although they gave virtually no credit to the Jesuits for their innovation, the Ch'ien-lung compilers broke new ground by placing mathematics and astronomy under the same framework. The editorial overview for the "Astronomy and Mathematics" section of the *General Catalog* represented a native response to the Jesuit impact on "new methods" (*hsin fa* 心法) that had successfully enabled more mathematically precise calculations of the calendar. Moreover the pride the editors took in acknowledging how Ch'ing scholars had balanced and unified "Western and Chinese" mathematics in the process of recovering the "single unknown" techniques made it clear that they regarded the "Chinese origins of Western Learning" as the key ingredient for overcoming the less informed mathematics of their Ming predecessors.[40]

## Juan Yüan and the *Biographies of Mathematical Astronomers*

Juan Yüan's assemblage of a staff to compile the *Biographies of*

38 SKCSTM, *chüan* 106-107 ; Alexander Wylie's 1867 *Notes on Chinese Literature*, pp. 106-130, presents a catalog of Chinese works in the category of *T'ien wen suan fa*. See also, Elman, *From Philosophy to Philology*, pp. 202-204.

39 See the editorial introduction to the "Astronomy and Mathematics" classification in the SKCSTM, 106.1a-2a.

40 SKCSTM, 106.1b.

Table 1 Forty-four Subdivisions of the Imperial Library (*Ssu k'u chüan shu* 四庫全書). Fields associated with natural studies have been highlighted.

| | |
|---|---|
| Classics | History |
| **Change** | Dynastic Histories |
| Documents | Annals |
| **Poetry** | Topical Records |
| Rituals | Unofficial Histories |
| Spring & Autumn Annals | Miscellaneous Histories |
| Filial Piety | Official Documents |
| General Works | Biographies |
| Four Books | Historical Records |
| **Music** | Contemporary Records |
| Philology | **Chronography** |
| | **Geography** |
| | Official Registers |
| Masters | Institutions |
| Literati | Bibliographies and Epigraphy |
| Military Strategists | Historical Criticism |
| Legalists | |
| **Agriculturalists** | |
| **Medicine** | Literature |
| **Astronomy & Mathematics** | Elegies of Chu |
| **Calculating Arts** | Individual Collections |
| **Arts** | General Anthologies |
| **Repertories of formulas, recipes** | Literary Criticism |
| Miscellaneous Writers | Songs & Drama |
| **Encyclopedias** | |
| Novels | |
| Buddhism | |
| Taoism | |

*Astronomers and Mathematicians* (*Ch'ou jen chuan* 疇人傳) while serving as governor of Chekiang province in Hang-chou from 1797 to 1799 marked the climax of the celebration of natural studies within the Yangtze delta literati world of the late eighteenth century. Juan Yüan's technical interests were influential because of his status as a patron of Yangtze delta scholarship, particularly evidential scholars from Yang-chou. He had become famous among his peers in 1791 when his prose-poem for the Hanlin Academy special examination topic on early Yüan astronomy was singled out by the Ch'ien-lung emperor for special praise.[41]

Juan was aided in his Hang-chou project by many of the leading evidential scholars of the late Ch'ien-lung period: Li Jui, Ch'ien Ta-hsin, Chiao Hsun, Ling T'ing-k'an 凌廷堪 (1757-1809), T'an T'ai 談泰, among others. Their efforts in astronomy have been described by Nathan Sivin as "a programmatic synthesis of traditional and Western astronomy designed to encourage the study of the latter in order to improve the former. Juan and his co-editor Li Jui, emphasized the old idea that the roots of modern astronomy are to be found in ancient China." Their efforts reaffirmed the value of mathematics and astronomy as part of a classical education.



Juan did not include in the collection those associated with fortune telling and numerology (*shu shu* 數術), and he opposed connecting mathematical astronomy with musical theory or studies of the *Change Classic*. As a conservative, however, he was critical of three new findings introduced by Michel Benoist: (1) heaven and earth are round; (2) planets follow elliptical paths; and (3) the sun is stationary. Although Benoist had finally presented the "heliocentric" Copernican system to China, Juan Yüan found such views unacceptable, in part because they contradicted earlier Jesuit presentations of Copernicus, which had reduced the latter's views to the "geoheliocentric" Tychonic system. Juan sought a fusion of European and Chinese mathematics based on shared conceptions. For astronomy, he sought an accurate, predictive computational system that would be based on improved techniques, not Copernican cosmology. Juan's views were influential empire-wide because in 1799 Juan also served as director of the mathematics section of the Dynastic

41 Wang P'ing 王萍, "Juan Yüan yü 'Ch'ou jen chuan'" 阮元與疇人傳 (Juan Yüan and the Biographies of Mathematical Astronomers), *Chung yang yen chiu yüan chin tai shi yen chiu so chi k'an* 中央研究院近代史研究所集刊, v. 4 (1973), pp: 601-611.

School in Peking.[42]

Containing summaries of the works of 280 mathematicians and astronomers, including thirty-seven Europeans, the *Biographies* was followed by four supplements in the nineteenth century. The collection was reissued in 1829 with only Ch'ing biographies, and it was later enlarged and reprinted in 1849. In 1840, for example, Lo Shih-lin added 43 sections on Sung-Ch'ing mathematical astronomers based on new sources for the "single unknown" and "four unknowns" techniques recovered from the Sung and Yüan, such as *Yang Hui's Calculation Methods* and the *Jade Mirror of the Four Unknowns*. In 1857, Alexander Wylie worked with Wang T'ao 王韜 (1828-1897) to improve on the views presented in the collection, particularly critiquing the "Chinese origins" narrative.[43]

Interest in mathematical astronomy among literati, which had developed steadily since Mei Wen-ting, had grown in importance outside the imperial court by the late eighteenth century. This growth was tied to the popularity of evidential studies outside the patronage networks of the Manchu court, which had sponsored Manchu and Mongol bannermen to try to control such knowledge. By connecting mathematics and astronomy to classical studies, Juan Yüan successfully integrated mathematical astronomy with evidential studies. Because mathematics and natural studies remained dependent on classical studies, Juan Yüan and his colleagues revived the ancient category of calendricists (*ch'ou jen* 疇人), whom he now considered "mathematical astronomers."



In the mid-eighteenth century, the official *Ming History* had already described the ancient dispersion of classical calendricists (*ch'ou jen*) to the Western region (*Hsi yü* 西域), specifically to the Islamic world. Moreover, the term for classical " calendricist" (*ch'ou jen*) had been used in the canonical *Artificer's Record* (*K'ao kung chi* 考工記) and Ssu-ma Ch'ien's 司馬遷 *Records of the Official Historian* (*Shih chi* 史記) during the Han dynasty. We have seen above that earlier in the eighteenth century, the K'ang-hsi emperor had singled out "palace graduates in mathematical astronomy" (*ch'ou jen chin shih* 疇人進

42 Sivin, "Copernicus in China," pp. 45-50 ; Martzloff, *A History of Chinese Mathematics*, pp. 166-172.

43 Li and Du, *Chinese Mathematics*, pp. 232-233 ; *Ch'ien Pao-ts'ung k'o hsüeh shih lun wen hsüan chi*, pp. 308-309 ; Paul Cohen, *Between Tradition and Modernity: Wang T'ao and Reform in Late Ch'ing China* (Cambridge, Mass.: Council on East Asian Studies, Harvard University, 1987), pp. 176-177.

士) such as Wang Lan-sheng, Minggatu, and Mei Chüeh-ch'eng for special honors.

Such usage was now reworked by Juan Yüan and T'an T'ai in their lead accounts of the meaning and scope of "mathematical astronomers" in the *Biographies*. They employed the term as a classical sanction for a new intellectual and social category of contemporary scholar-literati such as Mei Wen-ting, Tai Chen, and Ch'ien Ta-hsin, which also referred to a genealogy of professionalized skills in mathematics and astronomy going back to antiquity. This orthodox term was the first of several used in the eighteenth and nineteenth centuries to describe European "scientists" in classical Chinese.[44]

The category for mathematical astronomer (*ch'ou jen*) also had its conceptual roots in mathematics as one of "six arts" (*liu i* 六藝) of ancient, aristocratic scholars. Beginning in the late Ming, as literati increasingly engaged in the study of mathematics and astronomy, two types of specialists emerged: (1) specialists in calendar making; (2) literati with an academic interest in mathematics. These two categories were most evident during the K'ang-hsi era when mathematical study, which was a required tool for calendrical reform, was upgraded from an trivial skill. The academic climate among evidential scholars, along with imperial patronage, helped make mathematics and astronomy a collateral branch of classical learning.[45]

Despite their distance from the court, literati who favored mathematical astronomy, such as Juan Yüan, never thought it might eventually gain an independent position from other fields of classical learning. As calendrical difficulties declined in importance during the eighteenth century due to the success of the Ming-Ch'ing astronomical reforms inspired by the Jesuits and their Chinese counterparts, mathematics emerged as an independent field of inquiry in evidential research and among Han Learning scholars, particularly in Yang-chou. As cultural and political disputes over the calendar declined, literati debates shifted in the early nineteenth century to acclaim the achievements of



44 Juan Yüan 阮元, "Ch'ou jen chuan fan li" 疇人傳凡例 (Conventions of the Biographies of mathematical astronomers), in Juan Yüan, *Ch'ou jen chuan* 疇人傳 (Biographies of mathematical astronomers) (Taipei: Shih chieh shu chü, 1962), pp. 1-5 ; T'an T'ai 談泰, "Ch'ou jen chieh" 疇人解 (Explanation of mathematical astronomers), in *Ch'ou jen chuan*, pp. 1-4.

45 Limin Bai, "Mathematical Study and Intellectual Transition in the Early and Mid-Qing," *Late Imperial China,* v. 16, no. 2 (December 1995), pp. 23-61.

native mathematics.[46]

**Literati Natural Studies: Classics and Mathematics**

Literati in the nineteenth century were not doomed by the premises of Ch'ing classicism to preclude interest in natural studies and mathematics before the Opium War. Lord Macartney had recognized the Chinese interest in astronomy, but he had erred when he reduced it to "astrological trifling, the goal of which is the calculation of auspicious times." Moreover, because he was unaware of Chinese expertise in mathematical astronomy, Macartney indicated that the Chinese had no notion of algebra and possessed only a limited understanding of geometry and plain trigonometry. Englishmen such as Macartney were continuing a tradition of denigrating Chinese natural studies by Europeans that had begun with the Jesuits.[47]

Despite their textual focus, eighteenth century evidential scholars successfully restored a place for mathematical studies within the framework of the Chinese origins of Western learning. Even the massive *Ch'ing Exegesis of the Classics* (*Huang Ch'ing ching chieh* 皇清經解), a scholarly collection published in the early nineteenth century and devoted exclusively to evidential scholarship, included a significant number of works on natural studies and mathematical astronomy. Because it was the first comprehensive collection of Ch'ing contributions to classical scholarship, the 1829 publication of the *Ch'ing Exegesis* in Kuang-chou, after four years of compiling and editing at the Sea of Learning Hall (*hsüeh hai t'ang* 學海堂) under the auspices of Juan Yüan, then governor-general there, was greeted with acclaim in China, Korea, and Japan.[48]

An imposing anthology of some 180 diverse works by 75 seventeenth and eighteenth century authors in more than 360 volumes totaling some 1,400 chapters (*chüan*), the *Ch'ing Exegesis of the Classics* represented a major tribute to the research carried out by evidential scholars. It served as a collection of exemplary works from the Yangtze delta community in the seventeenth and

46 Pingyi Chu, "Western Astronomy and Evidential Study: Tai Chen on Astronomy," in Yung Sik Kim and Francesca Bray, eds., *Current Perspectives in the History of Science in East Asia* (Seoul: Seoul National University Press, 1999), p. 144.

47 George Macartney, *An Embassy to China; Being the Journal Kept by Lord Macartney During His Embassy to the Emperor Ch'ien-lung, 1793-1794*, edited with an introduction and notes by J. L. Cranmer-Byng (London: Longmans, 1962), p. 264.

48 Fujitsuka Chikashi, *Nichi Sen Shin no bunka kō ryū*, p. 108.

eighteenth centuries. Although a continuation to earlier commentaries and annotations of the Five Classics and Four Books, the *Ch'ing Exegesis* was also a response to early Ch'ing collectanea (*ts'ung-shu* 叢書) and encyclopedias that had stressed Ch'eng-Chu learning.[49]

What was noteworthy about Juan Yüan's summation of Ch'ing classical scholarship was not only its spotlight on the Han Learning and evidential studies produced by literati scholars from intellectual centers in the Yangtze delta. His partisan focus also allowed Juan to expand the scope for imperial classical studies beyond the domain of Sung Learning and Ch'eng-Chu studies. Via the accolades for Han Learning, Juan and his staff incorporated the works of many of the Ch'ing authors he had included in the *Biographies of Mathematical Astronomers*.

For example, Juan Yüan included verbatim major sections from the *Biographies* dealing with Ch'ing mathematical astronomers, beginning with two chapters on the mathematical astronomer Wang Hsi-ch'an 王錫闡 (1628-1682), followed by a section on Mei Wen-ting and an extensive chapter on Tai Chen. Remarkably, this was followed by four chapters from the *Biographies* on "Westerners," with mention of Greek scholars, including Aristotle and Ptolemy, and Europeans such as Copernicus, Tycho Brahe (1546-1601), Matteo Ricci, Adam Schall (1592-1666), Ferdinand Verbiest, and Nicolas Smogolenski (1611-1656). Isaac Newton was also briefly mentioned for his revision of Tycho Brahe's length of the solar year, which Juan et al. copied from the 1742 *Supplement to the Compendium of Observational and Computational Astronomy*. Inclusion of foreigners to this degree was unprecedented in a repository of Chinese classical learning. The era of eliding European astronomical scholarship, dominant since the Yung-cheng reign, was effectively redressed forty years before the onset of the Opium War.[50]

In addition, three works on the *Artificer's Record* (*K'ao kung chi*) chapter of the *Rituals of Chou* (*Chou li* 周禮) were incorporated in the *Ch'ing Exegesis*, including Tai Chen's and Cheng Yao-t'ien's 程瑤田 (1725-1814) illustrated studies of ancient ceremonial bronze bells. Juan also included his own



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49 See Elman, *From Philosophy to Philology*, pp. 126-133.

50 Juan Yüan, ed., *Huang Ch'ing ching chieh* 皇清經解 (Ch'ing exegesis of the Classics) (Reprint of the 1892 ed., Taipei: Fu-hsing shu chü, 1961), chapters 1059-1068, especially 1067.1a-b (v. 15, p. 11,324) on Newton.

*Explications Using Diagrams of the Design of Wheeled Carriages in the "Artificer's Record,"* his first published work. Similarly, Juan printed two important studies of the historical geography in the "Tributes of Yü" (*Ta Yü mo* 大禹謨) chapter of the *Documents Classic*. The lead work was Hu Wei's 胡渭 (1633-1714) highly praised *A Modest Approach to the Tributes of Yü* (*Yü kung ch'ui chih* 禹貢錐指), which corrected many mistakes in earlier commentaries. In addition, numerous geographical accounts and the chronology of events in the classics were published in Juan's collectanea.[51]

Sheng Pai-erh's 盛百二 (fl. ca. 1756) study of the astronomy in the *Documents Classic* was copied in the *Ch'ing Exegesis* even though Sheng was not known for any other significant works, though he had taught in academies for over a decade. Sheng's expertise in astronomy and trigonometry had influenced many of his students. His work was filled with illustrations based on the Tychonic geoheliocentric system from the late K'ang-hsi *Compendium of Observational and Computational Astronomy*. It also had many references to Jesuit scholarship on star maps, eclipses, and the motion of the planets.[52]

Juan Yüan added Ch'en Mao-ling's 陳懋齡 1797 *Examination of Mathematics and Astronomy in Classical Works* (*Ching shu suan hsüeh t'ien wen k'ao* 經書算學天文考) to the collection. Ch'en had become interested in Mei Wen-ting's work in 1793 and subsequently inquired about Western learning to a European in Kuang-chou who was on his way to Peking to serve in the Astro-calendric Bureau. Ch'en's own inquiry stressed that the *Computational Methods in Nine Chapters* classic represented the origins of remainder problems, "single unknown" procedures, and other calculation techniques.

Based on Juan Yüan's findings, for example, Ch'en affirmed the theory of a round earth. He then claimed that the geoheliocentric position, which he argued was first enunciated in the *Rituals of Chou*, proved that the earth was round, a position that Juan Yüan had rejected. In effect, Chen's *Examination* served as a repository of native mathematical astronomy that had been reinvigorated by the impact of Jesuit studies.[53]



51 See the table of contents to Juan Yüan, ed., *Huang Ch'ing ching chieh*, v. 1, pp. 9-32. See also Elman, *From Philosophy to Philology*, p. 243.

52 Juan Yüan, ed., *Huang Ch'ing ching chieh*, chapters 485-490 (v. 7, pp. 5305-5394), especially 488.2b-4b (pp. 5348-5349).

53 Juan Yüan, ed., *Huang Ch'ing ching chieh*, 1328.1a-2a (v. 19, p. 24459), especially 1328.22a-33a (pp. 14471-14472) ; *Ch'ou jen chuan*, v. 2, 48.634-637.

Many other straightforward classical works that also incorporated natural studies were included, but their technical content was hard to locate. To make the technical subjects in the *Ch'ing Exegesis* more accessible, the Han-lin academician Yü Yüeh 俞樾 (1821-1907) later prepared a topical index to Juan Yüan's collection as part of late Ch'ing efforts to document the Chinese priority in scientific knowledge at a time when modern science was being introduced by Protestant missionaries. Yü deemed this index important for his students at the "Refined Study for the Explication of the Classics" (*Ku ching ching she* 詁經經舍) Academy in Hang-chou where he taught for thirty years beginning in 1867. While governor of Che-chiang in 1801, Juan Yüan had established the "Refined Study" in Hang-chou to honor Later Han classicists and to link a classical education with a commitment to "concrete studies" (*shih hsüeh* 實學). Juan had seen to it that students there would be examined in astronomy, mathematics, and geography, in addition to their literary and textual studies.[54]



Yü Yüeh's index of topics in the *Ch'ing Exegesis* at first sight looked very much like the table of contents for a Ming-Ch'ing encyclopedia. It opened with the category of astronomy (*t'ien wen* 天文), which was tied to the use of mathematics to measure the heavens. The index went on to include 42 other categories ranging from human relations, morality, and political matters to rituals, food and drink, and things. "Things" were delimited to animals (with feathers or hair) and plants from grasses and vegetables to crops, trees and bamboo. Despite the similarity to earlier encyclopedias, however, the terminology Yü employed was drawn from a new era. What Yü called "astronomy" (*t'ien wen* 天文) was the earlier term for astrology. Instead of pharmacopoeia (*pen ts'ao* 本草), the index used the new term for plants (*chih wu* 植物) derived from the study of horticulture, which had been introduced through Protestant translations after the Opium War.[55]

## Mathematics among Literati in an Age of Evidential Research

In addition to the classics, we can also discern a more general interest in

54 Elman, "The Hsüeh-hai T'ang and the Rise of New Text Scholarship in Canton," *Ch'ing shih wen t'i* (now *Late Imperial China*), v. 4, no. 2 (December 1979), pp. 51-82.

55 Yü Yüeh's "Tzu-hsü" (Preface) to the 'Huang Ch'ing ching chieh chien mu' (Topical index to the Ch'ing exegesis of the classics) (1886 edition), pp. 1a-2a. My thanks to Ming-hui Hu for providing me with a copy of Yü's index. See also ECCP, pp. 944-945 ; Elman, *From Philosophy to Philology*, p. 162.

mathematics and natural studies in late Ch'ing novels. Casual references to some aspects of Chinese mathematics had been included in the late eighteenth century novel *A Country Codger's Words of Exposure* (*Yeh sou p'u yen* 野叟曝言, ca. 1780) by Hsia Ching-ch'ü 夏敬渠 (1705-1787). The topics of mathematics and natural studies were presented in far more detail, however, in the early nineteenth century novel *Flowers in the Mirror* (*Ching hua yüan* 鏡花緣, ca. 1821-1828) by Li Ju-chen 李汝珍 (ca. 1763-1830). There, mention of such technical subjects occurred in the form of chats and quizzes by women at parties held to celebrate their success in the civil examinations. Seven of the female scholars had some knowledge of mathematics.[56]

The fantasy of a realm of women knowledgeable in the classics and mathematics was prescient in the early nineteenth century, although Li Ju-chen's ending for the novel reaffirmed the literati world of his day. That women in the novel understood mathematics parodied an academic world in which most literati were still not well informed about natural studies. Those we have focused on above, like their counterparts in Europe, were exceptional. The larger questions *Flowers in the Mirror* raised about conventional concepts and practices, such as women's subordination to men and foot-binding, made Li Ju-chen's inclusion of mathematical puzzles in the novel both entertaining and enigmatic.[57]

Li Ju-chen can be tied to a circle of Yang-chou scholars who embodied the overlap between evidential studies and mathematics that he ascribed to women in the novel. Li's teacher was Ling T'ing-k'an, a Yang-chou scholar who was an acknowledged expert in mathematics and astronomy and had helped Juan Yüan compile the *Biographies of Astronomers and Mathematicians*. Ling had studied under Tai Chen and was influenced by Tai's works on mathematics such as the *Calculations Using Counting Rods* (*Ts'e suan* 測算), the *Record of Measuring Segments of a Circle by Computing the Sides of a Right-angled Triangle* (*Kou ku ko yüan chi* 勾股割圜記), as well as the mathematical classics Tai had retrieved from the Imperial Library.



56 Li Ju-chen 李汝珍, *Ching hua yüan* 鏡花緣 (*Flowers in the Mirror*) (Taipei: Hsüeh hai ch'u pan she, 1985), pp. 415-418, 484-492, 527-534 ; *Flowers in the Mirror, by Li Ju-chen* abridged translation by Lin Tai-yi (Berkeley, University of California Press, 1965), pp. 133-141, 229-235, 242-244. Compare Yu Wang Luen, "Knowledge of Mathematics and Science in Ching-Hua-Yuan," *Oriens Extremus*, v. 21, no. 2 (1974), pp. 217-236.

57 Li Ju-chen, *Ching hua yüan*, pp. 229-235. See also Maram Epstein, "Engendering Order: Structure, Gender, and Meaning in the Qing Novel Jinghua Yuan," *Chinese Literature: Essays, Articles, and Reviews*, v. 18 (December 1996), pp. 105-131.

Another Yang-chou native, Ch'eng Yao-t'ien, was linked to both Ling and Tai and had written two works on the *Chou Dynasty Classic of Gnomonic Computations* dealing with properties of the square. Chiao Hsün, another of Tai Chen's Yang-chou partisans, prepared several mathematical works on squares and explanations of mathematical processes. He had also completed a textbook entitled *General Explanations of Extracting Roots* (*K'ai fang t'ung shih* 開方通釋) and the more technical *An Explanation of Single Unknown Procedures* (*T'ien yüan i shih* 天元一釋). In addition, Li Ju-chen's brother-in-law, Hsu Kuei-lin 許桂林 (1778-1821), had prepared instructional manuals for "single unknown" procedures and arithmetic.[58]

Measurements of the circumference of a circle in Li Ju-chen's novel were based on the formula that the circumference equals the diameter times $\pi$. The value of $\pi$ that Li chose is interesting because some evidential scholars preferred a less accurate value from antiquity over a more accurate one from the Sung dynasty. The *Chou Dynasty Classic of Gnomonic Computations*, for example, gave the value of $\pi$ (*pie*) = "3," but since the Han dynasty there had been many efforts to obtain a more precise value. In his notes for the *Computational Methods in Nine Chapters* completed circa A.D. 263, Liu Hui inscribed an equilateral hexagon (with one side equal to the radius) in a circle to show that $\pi$ had to be greater than "3." Liu arrived at a lower value of "3.14" plus 64/625 by calculating the perimeter of a cyclic polygon of 96 sides inscribed in the circle. He also calculated the upper value of $\pi$ as "3.14" plus 169/625 by inscribing a circle inside a touching polygon. The real value was between these two limits.

In medieval times, Tsu Ch'ung-chih 祖沖之 (429-500) had arrived at a more accurate calculation by obtaining a range for $\pi$ between "3.1415927" and "3.1415926." Subsequently the intermediate figure of 3.14159265 was calculated. Both the Sui and Chin dynasty treatises on the medieval calendar had used the calculation of "3.14159265" that Tsu had made. In the late K'ang-hsi era, the *Collected Basic Principles of Mathematics* gave "3.141592653" as the value of $\pi$, while Chu Hung 朱鴻 gave the value to 39 decimal points later in the eighteenth century. Unlike evidential scholars such as Ch'ien Ta-hsin and Li Jui who settled for a value of $\pi$ = "3.16," Li Ju-chen used the figure of "3.14"

58 Lin Tai-yi, "Introduction," *Flowers in the Mirror, by Li Ju-chen*, pp. 5-9 ; Luen, "Knowledge of Mathematics," pp. 235-236.

for the novel, which was based on Tsu's calculations.[59]

Aspects of the natural world were also discussed by women in *Flowers in the Mirror*. To determine the weight of matter and the velocity of sound, for instance, Li Ju-chen presented the problem in light of the velocity of sound through the medium of lightning and thunder. The speed the novel gave was 1,285.7 feet (*ch'ih* 尺) per second. Newton had given a theoretical value of 979 feet per second and experimental value of 1,142 feet per second for the speed of sound. It is likely that Li Ju-chen's figure was derived from the *Collected Basic Principles of Mathematics* (*Shu li ching yün*), where the solution to the velocity of sound of a canon blast was given as 1,285.7 feet (*ch'ih*) per second. Converted to meters, the velocity was about 397.3 meters per second, about 20% higher than the modern value.[60]

The level of knowledge of mathematics and natural studies in Li Ju-chen's novel reflected the academic climate among evidential scholars, which had helped make mathematics and astronomy an important part of classical learning. Through the K'ang-hsi emperor's promotion of the *Collected Basic Principles of Mathematics* in the early eighteenth century, the Jesuit introduction of European mathematics and astronomy was taken seriously by leading classical scholars. Despite the Yung-cheng emperor's closed door policy from 1723, which helped impede the transmission of eighteenth century science in Europe, the revival of native mathematics in the eighteenth century was noteworthy.



Tai Chen, for example, insisted on deferring to the native tradition when using foreign knowledge. Tai maintained that the essential elements of astronomy and mathematics could be located in ancient classical texts. If studied properly, according to Tai, the Classics would prove themselves repositories of mathematical and astronomical knowledge that had been lost due to neglect and lack of understanding. He set out, for instance, to show--mistakenly as it turned out--that a cryptic passage in the *Documents Classic* revealed that the ancients had been aware of the complicated path of the sun on the celestial sphere. In the process, Tai concluded that this and other examples "are clear proof that Western methods were derived from the *Chou Dynasty Canon of Gnomonic Computations* (*Chou pi suan ching*)," which he had recovered from the Ming archives.[61]

59 Li Ju-chen, *Ching hua yüan*, pp. 531-532 ; *Flowers in the Mirror, by Li Ju-chen*, pp. 243 ; Luen, "Knowledge of Mathematics," pp. 221-224.

60 Li Ju-chen, *Ching hua yüan*, pp. 533-534 ; Luen, "Knowledge of Mathematics," pp. 233-234.

61 Elman, *From Philosophy to Philology*, pp. 118-119.

Similarly, other evidential scholars also valued the recovery of ancient mathematical texts. Li Jui's biography of Ch'ien T'ang 錢塘 (1735-1790) in the *Biographies of Astronomers and Mathematicians*, for instance, cited Ch'ien's confirmation that the ancient value of $\pi$ had been "3.16," and that this was more acceptable than Liu Hui's and Tsu Ch'ung-chih's more accurate--but later--finding of "3.14." That Li Jui and Ch'ien Ta-hsin both upheld the more ancient finding indicates that many evidential scholars skewed their "search for the truth from facts" (*shih shih ch'iu shih* 實事求是) in light of ancient learning.

In this way, the revival of ancient mathematics precluded the development of mathematics itself. On the other hand, Wang Lai, preferred to use the European mathematical notation from the Imperial Observatory, for which he was condemned, and Tung Yu-ch'eng was critical of Ch'ien T'ang's affirmation of $\pi$ = "3.16" and affirmed Liu Hui's value of "3.14" by referring to the Jesuit inspired *Collected Basic Principles of Mathematics*. Wang Lai ironically noted: "Contemporary philologists always focus on what their predecessors did, and do nothing but copy what has already been written. They are never able to discover what their predecessors had not yet discovered."[62]



The literary focus of Ch'ing scholarly interests in mathematics and astronomy did not prevent scholars such as Tai Chen and Ch'ien Ta-hsin from concentrating on mathematics and astronomy, but they connected such studies to what they considered was their more central objective: the reconstruction of antiquity. Their lesser concern for new discoveries in mathematics prevented them from realizing the full potential of natural studies as an independent field of inquiry before the late nineteenth century. On the other hand, there were little contacts with Europeans after 1750 until the 1793 Macartney mission, and even after that no Europeans during the French Revolution and the following Napoleonic era transmitted the new sciences and calculus in England and France to China.

Their concern with documents restricted eighteenth century evidential scholars to a textual focus, even if they occasionally made use of archaeological findings or carried out some astronomical investigation. Once keeping the calendar accurate was no longer an insurmountable technical problem, research

62 Juan Yüan, *Ch'ou jen chuan*, 42.545 ; Horng Wann-sheng, "Chinese Mathematics at the Turn of the 19th Century," in Cheng-hung Lin and Daiwie Fu, eds., *Philosophy and Conceptual History of Science in Taiwan* (Netherlands: Kluwer Academic Publishers, 1993), pp. 183-190, especially p. 186.

on ancient astronomy was valued mainly for its application to classical studies. The exact fields of astronomy and mathematics were rarely perceived as anything more than fields ancillary to more important classical and historical concerns. Inquiries into natural phenomena, for the most part, were still dependent on textual evidence and not experimentation.

**The Usefulness of Recovering Ancient Mathematics**

Evidential scholars in the eighteenth century were not doomed to a lack of curiosity about the natural world or mathematics, but the philological biases that dominated their scholarship did not independently support the nascent research and experimentation required in the step-by-step quantification of the natural world. In light of the important place mathematics and astronomy occupied in evidential research, however, we cannot assume that because a scientific revolution did not take place in China it could not have taken place. It is remarkable how quickly--not overnight to be sure--the Chinese people adapted to the needs of science and technology in the late nineteenth century. Politics and economics, not science or technology, made it seem as if little progress had been made before the Ch'ing defeat during the Sino-Japanese War of 1894-1895.



The triumph of modern science in China, as in Europe, required an intellectual transformation that would exceed the boundaries of textual scholarship, and social, economic, and political changes that would challenge the social pedigree of classical studies. Such intellectual and technological transformations began during the Newtonian century in Europe, a century before they were fully understood in China. European science in eighteenth century China was not built upon and developed partly because of faulty Jesuit transmission that failed to challenge native classicism or provide an academic alternative. The preeminent position of classical studies along with its historical focus, remained intact. That preeminence would be shaken in the aftermath of the Taiping Rebellion.[63]

Nevertheless, important mathematical research did take place, and the educational institutions required for precise scholarship were already in place when Protestant missionaries made their way to the China coast in the post-Napoleonic era. Moreover, the successful reconstruction of Sung-Yüan-Chin

63 For discussion, see Nathan Sivin, "Why the scientific revolution did not take place in China--or didn't it?" reprinted in Sivin, *Science in Ancient China: Researches and Reflections* (Aldershot, Great Britain: Variorum, 1995), VII, pp. 45-66.

mathematical texts made it easier for a significant number of Chinese literati in the nineteenth century to recognize the precise significance of and the need to master the new developments in advanced algebra, analytic geometry, and the differential and integral calculus that would be introduced via translation by Protestant missionaries. These translations were possible because of the cooperation of "traditional" Chinese mathematicians who fully understood the "single unknown" and "four unknowns" techniques that had been successfully restored.

In the early nineteenth century, for example, Lo Shih-lin commented on the relative strengths of traditional and European mathematics before the impact of the calculus in China. Lo was versed in European mathematics after he studied in the Astro-calendric Bureau as a stipend student for seven years. His early work followed the Bureau's European tradition, but he changed his mind when he went to capital in 1822 for the provincial examination, which he never passed. While in Peking he had been finally able to read Chu Shih-chieh's *Jade Mirror of the Four Unknowns* on "single unknown" methods. In Chu's work he discovered a method powerful enough to solve sophisticated mathematical problems.

Lo, who perished in the Taiping assault on Yang-chou in 1853, pointed out that European mathematics--in terms of trigonometry, logarithms, and its method of borrowing roots and powers--was not as powerful as Sung-Yüan "single unknown" and "four unknowns" techniques, by then completely reconstructed, which could solve problems that Jesuit algebra could not. Lo successfully explored the geometrical properties of the cone using "single unknown" operations, and his 1840 *Supplement to the Calculations of Segments of Circles* (*Hu shih suan shu pu* 弧矢算術補) extended Li Jui's work on arcs by adding many problems Lo solved using "single unknown" procedures. Not yet aware of the calculus, he urged Chinese scholars not to follow European mathematics too closely for their practical measurements.[64]

With the introduction of the differential and integral calculus in the mid-nineteenth century, for which the Chinese tried bu could not find an ancient, native precedent, Li Shan-lan and other Chinese mathematicians such as Hua Hengfang admitted that although the "four unknowns" notation was perhaps superior to Jesuit algebra, which Alexander Wylie acknowledged, the Chinese

64 Horng, "Chinese Mathematics at the Turn of the 19th Century," pp. 187, 199 ; Hu Mingjie, "Merging Chinese and Western Mathematics," pp. 214-223 ; ECCP, pp. 538-539 ; Wylie, *Notes on Chinese Literature*, pp. 125.

had never developed anything resembling the calculus. Moreover, after the Opium War the most influential Chinese mathematicians no longer were devoted exclusively to the revival of ancient Chinese mathematics. They merged European and Chinese mathematics into a new synthesis, which drew extensively on the evidential studies of mathematics during the Ch'ien-lung era.